Topic Study Group 22

Mathematical Applications and Modelling in Mathematics Education

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ABSTRACT In the field of applications and mathematical modelling there is intensive research. Within the TSG, we have thematically addressed the teaching of mathematical modelling, teacher education, and modelling processes and competencies of school and university students. Future research directions are expected to consider theory building, empirical studies, including developing standardized research instruments, and the use of technology.

Keywords: Mathematical modelling; Teaching modelling; Teacher education; Modelling competencies and processes.

1. Theme and Description

The teaching and learning of mathematical applications and modelling is a world-renowned field of research in mathematics education and it has been an important theme for teachers and researchers especially during the last 50 years; and the importance has been growing worldwide during the last decade. This is evident, for example, in the International Congress on Mathematical Education (ICME) regular topic study groups and lectures on applications and modelling, and the series of conferences of the International Community on the Teaching of Mathematical Modelling and Applications (ICTMA) since 1983. This increasing interest is a consequence of several factors; on the one hand there is the public demand for the relevance of mathematics outside the discipline, and on the other hand there is an increasing number of research projects and empirical studies which focus on specific aspects of applications and modelling in mathematics teaching and learning. Many recent qualitative and quantitative research studies on mathematical modelling in school and higher education have focused on students and their modelling processes; however, teachers clearly play an important role in implementing mathematical modelling into mathematics lessons and in fostering students modelling competencies. Furthermore, classroom settings also play an important role. Enriching the focus on teacher practice in proposing and implementing interventional activities, there has been

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a research approach to the design of single modelling lessons as well as to the whole modelling learning environments at different school levels.

This topic study group (TSG-22) considers the importance of exploring relations between mathematics and the real world that occur in educational environments. It also recognizes the value of examining the discussions in research and development on the applications and modelling issues at the primary, secondary and tertiary school levels, including the mathematics teacher education. The TSG also recognizes the interplay between research and development of modelling learning environments (Greefrath et al. 2023). .

2. Program Overview

2.1. Team and participants

The TSG-22 team was composed of Xiaoli Lu (China), George Ekol (South Africa), Susana Carreira (Portugal, Co-Chair) and Gilbert Greefrath (Germany, Chair). From the large number of submissions, 10 long oral, 26 short oral and 8 posters were presented during the conference. The first authors of these papers and posters came from 20 different countries. The countries most strongly represented in TSG-22 were Chile, China, Germany and Japan.

2.2. Structure of the sessions

For the TSG, four sessions of 90 to 120 minutes each were available. Each session was chaired by one of the team members. The sessions were structured thematically. During the session, long and short oral contributions alternated. For some presentations, a joint discussion took place together, as far as the high number of contributions allowed. In the first session, in addition to the thematic part, there was an introduction for the group and a presentation as a thematic overview of the current state of research.

2.3. Theme 1: introduction and teaching mathematical modelling

Following a welcome and overview of sessions by the chair, Gabriele Kaiser made a long oral presentation on *The Teaching and Learning of Mathematical Modelling. A Description of the Current State-of-the-Art*. In particular, she addressed theoretical perspectives (Kaiser & Sriraman, 2006) and modelling competencies (Niss & Blum, 2020). This was a very relevant starting point for the work of the TSG. Subsequently, the *teaching of mathematical modelling* was examined from different angles. For example, sociocultural and geographical aspects were discussed, but also specifics of statistical modelling were highlighted (see Tab. 1).

Tab. 1. Presentations on the themes "introduction and teaching mathematical modelling"

Paper and author(s)	
[1]	The teaching and learning of mathematical modelling. a description of the current state-of-the-art. ***Gabriele Kaiser*** (Germany).
[2]	Sociocultural influences on mathematical modelling: an ethnomathematical perspective. ***Milton Rosa*** *and Daniel Clark Orey* (Brazil).
[3]	Teaching methods for modelling problems. ***Stanislaw Schukajlow*** *and* ***Werner Blum*** (Germany).
[4]	Examining the geographical features of the nasu area. analysing the origin of the nasu area using mathematics. ***Masahiro Takizawa*** (Japan).
[5]	A mathematical modelling technique as tool for teaching mathematics. ***Eloisa Benitez-Mariño*** (Mexico).
[6]	Theorizing tensions between mathematical modelling processes and conventional mathematics instruction. ***Wenmin Zhao*** *and Samuel Otten* (China).
[7]	The rationales of statistical modelling in education research from a mathematical modelling perspective. ***Takashi Kawakami*** *and Jonas Bergman Arleback* (Japan).
[8]	Modelling in a teacher education programme. ***Dragana Martinovic*** (Canada).

2.4. Theme 2: teacher education

The second topic of the TSG was teacher education. On the one hand, there were contributions on specific topics such as global warming or subject areas like STEM. On the other hand, there were contributions to promote certain aspects of professional competencies such as noticing skills or self-efficacy. Different types of school levels were taken into account (see Tab. 2).

Tab. 2. Presentations on the theme "teacher education"

Paper and author(s)	
[9]	Mathematical modelling in STEM contexts. Characterization of STEM skills and gender gaps in initial formation of mathematics teachers. ***María Aravena Diaz****, Marcelo Alejandro Rodriguez, Susan Valeria Sanhueza Henriquez, Maria Jose Seckel, and Angelica Urrutia Seplveda* (Chile).
[10]	Using assessment for learning to support students modelling activities. ***George Ekol*** (South Africa).
[11]	Epistemic states of university mathematics teachers in mathematical modelling education. ***George Gotoh****, Mitsuru Kawazoe and Hirofumi Ochiai* (Japan).
[12]	Using staged videos to foster pre-service teachers noticing skills. ***Alina Alwast*** *and Katrin Vorhölter* (Germany).
[13]	Prospective teachers self-efficacy for teaching mathematical modelling. ***Hans-Stefan Siller****,* ***Gilbert Greefrath****, Raphael Wess, and Heiner Klock* (Germany).
[14]	Pedagogy that supports mathematical modelling. One elementary school teachers story. ***Rejoice Akapame*** *and Robin Angotti* (USA).
[15]	Pre-Service mathematics teachers project-based mathematical modelling instruction: conception, task design, and enactment. ***JooYoung Park*** (USA).
[16]	The development of a modelling teacher education program starting from the transformation of a mathematised task into modelling tasks. ***Akihiko Saeki****, Masafumi Kaneko, Takashi Kawakami, and Toshikazu Ikeda* (Japan).
[17]	Prospective teachers of mathematics suspend common sense in solving word problem. ***Abolfazl Rafiepour*** *and Zohreh Khazaei* (Iran).

2.5. Theme 3: students modelling processes

The topic with the most contributions was on modelling processes and modelling competencies of students. Here the particular situations in different countries were considered and also different age groups were considered. Also, different ways of measuring modelling competencies were discussed. Various models for describing modelling processes were also discussed here, and the use of technology in modelling was explored (see Tab. 3).

Tab. 3. Presentations on the theme "students modelling processes"

Paper and author(s)	
[18]	The mathematical modelling landscape: a literature review on perspectives, methodology, content, unit of analysis, and geography. ***Armando Paulino Preciado Babb***, *Fredy Peña Acuña, Andrea Ortiz Rocha, and Armando Solares Rojas* (Canada).
[19]	Distinguishing the distinctions: observing the solving of a mathematical modelling task. ***Paola Andrea Ramirez Gonzalez*** (Chile).
[20]	Mathematical modelling skills of secondary students. ***Kwan Eu Leong*** (Malaysia).
[21]	Mathematical modelling in the new curriculum: are chinese students ready? ***Jian Huang*** *and Binyan Xu* (China).
[22]	Student presentations of mathematical modelling as a site for fostering reflective discourse. ***Hyunyi Jung***, *Corey Edison Brady, Jeffrey Allen McLean* (USA), *Angeles Dominguez* (Mexico), *and Aran Glancy* (USA).
[23]	How do undergraduate students hold the individual assumptions in collaborative modelling? ***Kazuhiko Imai*** *and Akio Matsazaki* (Japan).
[24]	Investigating students data moves in a citizen science based data-rich model-eliciting activity. ***Jeffrey Allen McLean***, *Corey Edison Brady, Hyunyi Jung, Aran Glancy* (USA), *and Angeles Dominguez* (Mexico).
[25]	Differences in students conceptions about mathematics when participating in a mathematical modelling contest. ***Flavio Guiñez*** (Chile).
[26]	Measurement mathematical modelling competency and its relationship to mathematical interests of seventh grade. ***Zhiyong Xie***, *Yaling Li, Tian Wang, and Jian Liu* (China).
[27]	Assessment of four-grade students mathematical modelling competency: take one city of china as an example. ***Tian Wang***, *Zhiyong Xie, and Jian Liu* (China).
[28]	Study of a problem solving using the extended mathematical working space framework. ***Laurent Moutet*** (France).
[29]	Introducing a composite model for investigation in real world problem. ***Kazem Abdollahpour Chenary*** *and* ***Abolfazl Rafiepour*** (Iran).
[30]	A computer-based learning environment on mathematical modelling: research design and pilot studies. ***Lena Frenken*** (Germany).

2.6. Theme 4: university students modelling processes

The fourth and last topic of the TSG was dealing with modelling processes and modelling competencies of students at the university. Various models for describing modelling processes were discussed and the use of technology in modelling at the university was considered. Also, different instruments for the assessment of modelling competences at the university were presented. It was also described how one can learn certain mathematical contents through mathematical modelling (see Tab. 4).

Tab. 4. Presentations on the theme “university students modelling processes”

Paper and author(s)
[31] Undergraduate students’ modelling routes mediated by technology in the learning of linear transformations. ***Susana Carreira***, *Guillermo Enrique Ramirez Montes, and Ana Claudia Henriques* (Portugal).
[32] Is quality teaching favourable for the development of modelling competency? an empirical study with engineering students over two years. ***Rina Durandt***, *Werner Blum, and Alfred Lindl* (South Africa).
[33] Validating a modelling competencies assessment. ***Jennifer A. Czocher***, *Sindura Kandasamy, and Elizabeth Roan* (USA).
[34] Mathematical modelling with biology undergraduates: using activity theory to understand tensions. ***Yuriy Rogovchenko*** (Norway).
[35] Calculus learning competency through mathematical modelling. ***Lorenza Illanes*** *and* ***Roberto Retes*** (Chile).
[36] Research on evaluation of college students' mathematical modelling ability based on AHP and BP neural network. ***Yixin Dong***, *Huanhuan Zhang, Meng Ci, and Ziyi Wang* (China).

3. Future Directions and Suggestions

Even though mathematical modelling in mathematics education is currently being very intensively discussed and researched, there are still some open questions for the future. These relate to both theoretical areas and empirical areas. For example, the further development of central constructs such as modelling competencies or professional competencies for teaching modelling is an interesting field of consistent research development. For further empirical studies, appropriate standardized test instruments are needed, which should be developed and shared in the community. This could also contribute to a better interlinking of studies on the methodological and thematic side. Furthermore, the changes caused by a heterogeneous school population and the use of technology in all school levels up to university should be responded to accordingly.

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